MONTROSE & OLATHE SCHOOLS

Math/Algebra 1 2016-2017

Instructional Calendar

International Society for Technology in Education Standards

1. **Creativity and innovation:** Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
2. **Communication and collaboration**: Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
3. **Research and information fluency**: Students apply digital tools to gather, evaluate, and use information.
4. **Critical thinking, problem solving, and decision making**: Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
5. **Digital citizenship**: Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.
6. **Technology Operations and Concepts:** Students demonstrate and sound understanding of technology concepts, systems and operations.

Quarter 1

Standard 1 (Number Sense, Properties and Operations)	Standard 2 (Patterns, Functions, and Algebraic Structures)
• Use units to understand and solve application problems • Introduce dimensional analysis and unit conversions • Choose scales and units appropriately for models • Recognize that certain functions are more appropriate for particular situations than others	• Investigate the Growth of Patterns • Investigate Graphs of Quadratic Functions, Graph using x/y table (x values given) • Identify Vertex, Axis of symmetry, x and y intercepts, positive or negative parabola • Investigate Graphs of Cube Root Functions, Graph using x/y table (x values given) • Introduce Function Machines • Evaluate functions, use function notation • Understand domain and range of varying functions • Growth of linear relationships • Slope formula, delta y/delta x, rise/run, rate of change • Y-Intercept and y=mx+b equations • Finding rate of change in word problems

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Quarter 2

Standard 2 Patterns, Functions, and Algebraic Structures	Standard 3 (Data Analysis, Statistics, and Probability)
● Reinforce the concept of slope/growth rate in context/word problems ● Develop a linear equation from a situation ● Find the equation of a line in slope-intercept form given: ○ slope and a point ○ two points ○ graph ○ table ● Simplify exponential expressions via expansion and exponent laws ● Represent equations physically and visually using algebra tiles ● Simplify expressions and solve equations using algebra tiles ● Write polynomials as a product and sum using area models (algebra tiles) ● Use generic rectangles and the distributive property to multiply expressions ● Solve one-variable equations with multiplication and absolute value ● Solve multivariable equations for one variable ● Change the form of a linear equation from standard to slope-intercept	● Interpret the slope and intercept of a linear model in a particular context

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Quarter 3		
Standard 1 (Number Sense, Properties and Operations)	Standard 2 Patterns, Functions, and Algebraic Structures	Standard 3 (Data Analysis, Statistics, and Probability)

● Write equations of word problems ● Choose the best strategy to solve a system of equations	● Solve systems of equations using… ○ Graphing ○ Equal Values ○ Substitution ○ Elimination ● One solution vs. infinite solutions vs. no solutions ● Find slope of best fit line ● Find y=mx+b equation of the best fit line	● Graph data points on a scatterplot ● Understand Correlation of data ● Choose points and draw a line of best fit ● Find residual values ● Interpreting correlation, slope, and y-int in context ● Introduction to curved best fit models ○ exponential growth and decay

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Quarter 4

Standard 1 (Number Sense, Properties and Operations)	Standard 2 Patterns, Functions, and Algebraic Structures	Standard 3 (Data Analysis, Statistics, and Probability)

	• Change quadratic equation written as a sum into its product form (factored form)
	• Factor quadratics with generic rectangles
	• Identify different forms of quadratics:
	  ○ Standard form
	  ○ Factored form
	  ○ Graphing (vertex) form
	• Factor quadratics with special cases: Difference of squares and GCF
	• Factor quadratics with diamond problems
	• Factor completely: GCF in combination with other methods
	• Special quadratics: difference of squares and sum of squares
	• Represent quadratic using table, graph, equation, situation
	• Write a quadratic equation from a situation
	• Zero product property
	• Solving quadratics using table, graph, equation
	• Solving quadratics using complete the square	

International Society for Technology in Education Standards

7. **Creativity and innovation:** Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
8. **Communication and collaboration**: Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
9. **Research and information fluency**: Students apply digital tools to gather, evaluate, and use information.
10. **Critical thinking, problem solving, and decision making**: Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
11. **Digital citizenship**: Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

12. **Technology Operations and Concepts:** Students demonstrate and sound understanding of technology concepts, systems and operations.

End of Year Expectations By Standard

Standard 1 (Number Sense, Properties and Operations)	Standard 2 Patterns, Functions, and Algebraic Structures	Standard 3 (Data Analysis, Statistics, and Probability)

• Use units to understand and solve application problems
• Utilize dimensional analysis and unit conversions
• Choose scales and units appropriately for models
• Choose appropriate limits for measurement of quantities
• Write equations of word problems
	• Solving multi-step equations
	• Simplifying expressions
	  ○ distributive property
	  ○ combining like terms
	  ○ eliminating fractions using LCD
	• basic factoring
	  ○ quadratic expressions with leading coefficient= 1
	  ○ polynomials using GCF
	• graphing linear equations in slope-intercept form and standard form
		• Interpret the slope and intercept of a linear model in a particular context given:
		  ○ situation
		  ○ situation and equation
		  ○ situation and graph

Content Area: Mathematics

Grade Level Expectations: High School

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates: *(Click on a Prepared Graduate Competency to View Articulated Expectations)*

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Concepts and skills students master:

1. The complex number system includes real numbers and imaginary numbers

Evidence Outcomes	21st Century Skill and Readiness Competencies
Students Can: a. Extend the properties of exponents to rational exponents. (CCSS: N-RN) i. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.[1] (CCSS: N-RN.1) ii. Rewrite expressions involving radicals and rational exponents using the properties of exponents. (CCSS: N-RN.2) b. Use properties of rational and irrational numbers. (CCSS: N-RN) i. Explain why the sum or product of two rational numbers is rational. (CCSS: N-RN.3) ii. Explain why the sum of a rational number and an irrational number is irrational. (CCSS: N-RN.3) iii. Explain why the product of a nonzero rational number	**Inquiry Questions:** 1. When you extend to a new number systems (e.g., from integers to rational numbers and from rational numbers to real numbers), what properties apply to the extended number system? 2. Are there more complex numbers than real numbers? 3. What is a number system? 4. Why are complex numbers important? **Relevance & Application:** 1. Complex numbers have applications in fields such as chaos theory and fractals. The familiar image of the Mandelbrot fractal is the Mandelbrot set graphed on the complex plane. **Nature Of:**

and an irrational number is irrational. (CCSS: N-RN.3) c. Perform arithmetic operations with complex numbers. (CCSS: N-CN) i. Define the complex number i such that $i^2=-1$, and show that every complex number has the form a + bi where a and b are real numbers. (CCSS: N-CN.1) ii. Use the relation $i^2=-1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. (CCSS: N-CN.2) d. Use complex numbers in polynomial identities and equations. (CCSS: N-CN) i. Solve quadratic equations with real coefficients that have complex solutions. (CCSS: N-CN.7)	1. Mathematicians build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. 2. Mathematics involves making and testing conjectures, generalizing results, and making connections among ideas, strategies, and solutions. 3. Mathematicians look for and make use of structure. (MP) 4. Mathematicians look for and express regularity in repeated reasoning. (MP)

[1] For example, we define $5^{\frac{1}{3}}$ to be the cube root of 5 because we want $(5^{\frac{1}{3}})^3=5^{(\frac{1}{3})3}$ to hold, so $(5^{\frac{1}{3}})^3$ must equal 5. (CCSS: N-RN.1)

Prepared Graduates: *(Click on a Prepared Graduate Competency to View Articulated Expectations)*

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Concepts and skills students master:

2. Quantitative reasoning is used to make sense of quantities and their relationships in problem situations

Evidence Outcomes	**21st Century Skill and Readiness Competencies**

Students Can:	Inquiry Questions:
a. Reason quantitatively and use units to solve problems (CCSS: N-Q) i. Use units as a way to understand problems and to guide the solution of multi-step problems. (CCSS: N-Q.1) 1. Choose and interpret units consistently in formulas. (CCSS: N-Q.1) 2. Choose and interpret the scale and the origin in graphs and data displays. (CCSS: N-Q.1) ii. Define appropriate quantities for the purpose of descriptive modeling. (CCSS: N-Q.2) iii. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. (CCSS: N-Q.3) iv. Describe factors affecting take-home pay and calculate the impact (PFL) v. Design and use a budget, including income (net take-home pay) and expenses (mortgage, car loans, and living expenses) to demonstrate how living within your means is essential for a secure financial future (PFL)	1. Can numbers ever be too big or too small to be useful? 2. How much money is enough for retirement? (PFL) 3. What is the return on investment of post-secondary educational opportunities? (PFL) **Relevance & Application:** 1. The choice of the appropriate measurement tool meets the precision requirements of the measurement task. For example, using a caliper for the manufacture of brake discs or a tape measure for pant size. 2. The reading, interpreting, and writing of numbers in scientific notation with and without technology is used extensively in the natural sciences such as representing large or small quantities such as speed of light, distance to other planets, distance between stars, the diameter of a cell, and size of a micro–organism. 3. Fluency with computation and estimation allows individuals to analyze aspects of personal finance, such as calculating a monthly budget, estimating the amount left in a checking account, making informed purchase decisions, and computing a probable paycheck given a wage (or salary), tax tables, and other deduction schedules. **Nature Of:** 1. Using mathematics to solve a problem requires choosing what mathematics to use; making simplifying assumptions, estimates, or approximations; computing; and checking to see whether the solution makes sense. 2. Mathematicians reason abstractly and quantitatively. (MP) 3. Mathematicians attend to precision. (MP)

Content Area: Mathematics

Grade Level Expectations: High School

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates: *(Click on a Prepared Graduate Competency to View Articulated Expectations)*

- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data

Concepts and skills students master:

1. Functions model situations where one quantity determines another and can be represented algebraically, graphically, and using tables

Evidence Outcomes	21st Century Skill and Readiness Competencies
Students Can: a. Formulate the concept of a function and use function notation. (CCSS: F-IF) i. Explain that a function is a correspondence from one set (called the domain) to another set (called the range) that assigns to each element of the domain exactly one element of the range.[1] (CCSS: F-IF.1)	**Inquiry Questions:** 1. Why are relations and functions represented in multiple ways? 2. How can a table, graph, and function notation be used to explain how one function family is different from and/or similar to another? 3. What is an inverse? 4. How is “inverse function” most likely related to addition and subtraction being inverse operations and to multiplication and division being inverse operations? 5. How are patterns and functions similar and different? 6. How could you visualize a function with four variables, such as

ii. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. (CCSS: F-IF.2)

iii. Demonstrate that sequences are functions,[2] sometimes defined recursively, whose domain is a subset of the integers. (CCSS: F-IF.3)

b. Interpret functions that arise in applications in terms of the context. (CCSS: F-IF)

i. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features[3] given a verbal description of the relationship. * (CCSS: F-IF.4)

ii. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.[4] * (CCSS: F-IF.5)

iii. Calculate and interpret the average rate of change[5] of a function over a specified interval. Estimate the rate of change from a graph.* (CCSS: F-IF.6)

c. Analyze functions using different

$x^2+y^2+z^2+w^2=1$?

7. Why couldn't people build skyscrapers without using functions?
8. How do symbolic transformations affect an equation, inequality, or expression?

Relevance & Application:

1. Knowledge of how to interpret rate of change of a function allows investigation of rate of return and time on the value of investments. (PFL)
2. Comprehension of rate of change of a function is important preparation for the study of calculus.
3. The ability to analyze a function for the intercepts, asymptotes, domain, range, and local and global behavior provides insights into the situations modeled by the function. For example, epidemiologists could compare the rate of flu infection among people who received flu shots to the rate of flu infection among people who did not receive a flu shot to gain insight into the effectiveness of the flu shot.
4. The exploration of multiple representations of functions develops a deeper understanding of the relationship between the variables in the function.
5. The understanding of the relationship between variables in a function allows people to use functions to model relationships in the real world such as compound interest, population growth and decay, projectile motion, or payment plans.
6. Comprehension of slope, intercepts, and common forms of linear equations allows easy retrieval of information from linear models such as rate of growth or decrease, an initial charge for services, speed of an object, or the beginning balance of an account.
7. Understanding sequences is important preparation for calculus. Sequences can be used to represent functions including e^x, e^{x^2}, sin x, and cos x.

representations. (CCSS: F-IF) i. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. * (CCSS: F-IF.7) ii. Graph linear and quadratic functions and show intercepts, maxima, and minima. (CCSS: F-IF.7a) iii. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. (CCSS: F-IF.7b) iv. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. (CCSS: F-IF.7c) v. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. (CCSS: F-IF.7e) vi. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. (CCSS: F-IF.8)	**Nature Of:** 1. Mathematicians use multiple representations of functions to explore the properties of functions and the properties of families of functions. 2. Mathematicians model with mathematics. (MP) 3. Mathematicians use appropriate tools strategically. (MP) 4. Mathematicians look for and make use of structure. (MP)

1. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (CCSS: F-IF.8a) 2. Use the properties of exponents to interpret expressions for exponential functions. (CCSS: F-IF.8b) 3. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (CCSS: F-IF.9) d. Build a function that models a relationship between two quantities. (CCSS: F-BF) i. Write a function that describes a relationship between two quantities.* (CCSS: F-BF.1) 1. Determine an explicit expression, a recursive process, or steps for calculation from a context. (CCSS: F-BF.1a)	

2. Combine standard function types using arithmetic operations.[8] (CCSS: F-BF.1b) ii. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. (CCSS: F-BF.2) e. Build new functions from existing functions. (CCSS: F-BF) i. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k, [9] and find the value of k given the graphs.[10] (CCSS: F-BF.3) ii. Experiment with cases and illustrate an explanation of the effects on the graph using technology. iii. Find inverse functions.[11] (CCSS: F-BF.4) f. Extend the domain of trigonometric functions using the unit circle. (CCSS: F-TF) i. Use radian measure of an angle as the length of the arc on the unit circle subtended by the angle. (CCSS: F-TF.1) ii. Explain how the unit circle in the	

coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. (CCSS: F-TF.2)	

[1] If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). (CCSS: F-IF.1)

[2] For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n $\geq$ 1. (CCSS: F-IF.3)

[3] Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (CCSS: F-IF.4)

[4] For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. (CCSS: F-IF.5)

[5] presented symbolically or as a table. (CCSS: F-IF.6)

[6] For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10,. (CCSS: F-IF.8b)

[7] For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. (CCSS: F-IF.9)

[8] For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. (CCSS: F-BF.1b)

[9] both positive and negative. (CCSS: F-BF.3)

[10] Include recognizing even and odd functions from their graphs and algebraic expressions for them. (CCSS: F-BF.3)

[11] Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.

For example, f(x) = $2x^3$ or f(x) = (x+1)/(x–1) for x ≠ 1. (CCSS: F-BF.4a)

Prepared Graduates: *(Click on a Prepared Graduate Competency to View Articulated Expectations)*

- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Concepts and skills students master:

2. Quantitative relationships in the real world can be modeled and solved using functions

Evidence Outcomes	**21st Century Skill and Readiness Competencies**
Students Can: a. Construct and compare linear, quadratic, and exponential models and solve problems. (CCSS: F-LE) i. Distinguish between situations that can be modeled with linear functions and with exponential	**Inquiry Questions:** 1. Why do we classify functions? 2. What phenomena can be modeled with particular functions? 3. Which financial applications can be modeled with exponential functions? Linear functions? (PFL) 4. What elementary function or functions best represent a given scatter plot of two-variable data?

functions. (CCSS: F-LE.1)

1. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. (CCSS: F-LE.1a)
2. Identify situations in which one quantity changes at a constant rate per unit interval relative to another. (CCSS: F-LE.1b)
3. Identify situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. (CCSS: F-LE.1c)

ii. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs.[12] (CCSS: F-LE.2)

iii. Use graphs and tables to describe that a quantity increasing exponentially eventually exceeds a quantity

5. How much would today's purchase cost tomorrow? (PFL)

Relevance & Application:

1. The understanding of the qualitative behavior of functions allows interpretation of the qualitative behavior of systems modeled by functions such as time-distance, population growth, decay, heat transfer, and temperature of the ocean versus depth.
2. The knowledge of how functions model real-world phenomena allows exploration and improved understanding of complex systems such as how population growth may affect the environment , how interest rates or inflation affect a personal budget, how stopping distance is related to reaction time and velocity, and how volume and temperature of a gas are related.
3. Biologists use polynomial curves to model the shapes of jaw bone fossils. They analyze the polynomials to find potential evolutionary relationships among the species.
4. Physicists use basic linear and quadratic functions to model the motion of projectiles.

Nature Of:

1. Mathematicians use their knowledge of functions to create accurate models of complex systems.
2. Mathematicians use models to better understand systems and make predictions about future systemic behavior.
3. Mathematicians reason abstractly and quantitatively. (MP)
4. Mathematicians construct viable arguments and critique the reasoning of others. (MP)
5. Mathematicians model with mathematics. (MP)

increasing linearly, quadratically, or (more generally) as a polynomial function. (CCSS: F-LE.3) iv. For exponential models, express as a logarithm the solution to $ab^{ct}=d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. (CCSS: F-LE.4) b. Interpret expressions for function in terms of the situation they model. (CCSS: F-LE) i. Interpret the parameters in a linear or exponential function in terms of a context. (CCSS: F-LE.5) c. Model periodic phenomena with trigonometric functions. (CCSS: F-TF) i. Choose the trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. * (CCSS: F-TF.5) d. Model personal financial situations i. Analyze the impact of interest rates on a personal financial plan (PFL) ii. Evaluate the costs and benefits of credit (PFL) iii. Analyze various lending	

sources, services, and financial institutions (PFL)	

[12] include reading these from a table. (CCSS: F-LE.2)

Prepared Graduates: *(Click on a Prepared Graduate Competency to View Articulated Expectations)* • Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations	
Concepts and skills students master: 3. Expressions can be represented in multiple, equivalent forms	
Evidence Outcomes	**21st Century Skill and Readiness Competencies**
Students Can: a. Interpret the structure of expressions.(CCSS: A-SSE) i. Interpret expressions that represent a quantity in terms of its context.* (CCSS: A-SSE.1) 1. Interpret parts of an expression, such as terms, factors, and coefficients. (CCSS: A-SSE.1a)	**Inquiry Questions:** 1. When is it appropriate to simplify expressions? 2. The ancient Greeks multiplied binomials and found the roots of quadratic equations without algebraic notation. How can this be done? **Relevance & Application:** 1. The simplification of algebraic expressions and solving equations are tools used to solve problems in science. Scientists represent relationships between variables by developing a formula and using values obtained from experimental measurements and algebraic manipulation to determine values of quantities that are difficult or impossible to measure directly such as acceleration due to gravity, speed of light, and mass of the earth.

2. Interpret complicated expressions by viewing one or more of their parts as a single entity.[13] (CCSS: A-SSE.1b) ii. Use the structure of an expression to identify ways to rewrite it.[14] (CCSS: A-SSE.2) b. Write expressions in equivalent forms to solve problems. (CCSS: A-SSE) i. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* (CCSS: A-SSE.3) 1. Factor a quadratic expression to reveal the zeros of the function it defines. (CCSS: A-SSE.3a) 2. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. (CCSS: A-SSE.3b) 3. Use the properties of	2. The manipulation of expressions and solving formulas are techniques used to solve problems in geometry such as finding the area of a circle, determining the volume of a sphere, calculating the surface area of a prism, and applying the Pythagorean Theorem. **Nature Of:** 1. Mathematicians abstract a problem by representing it as an equation. They travel between the concrete problem and the abstraction to gain insights and find solutions. 2. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 3. Mathematicians model with mathematics. (MP) 4. Mathematicians look for and express regularity in repeated reasoning. (MP)

exponents to transform expressions for exponential functions.[15] (CCSS: A-SSE.3c) ii. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. * (CCSS: A-SSE.4) c. Perform arithmetic operations on polynomials. (CCSS: A-APR) i. Explain that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (CCSS: A-APR.1) d. Understand the relationship between zeros and factors of polynomials. (CCSS: A-APR) i. State and apply the Remainder Theorem.[17](CCSS: A-APR.2) ii. Identify zeros of polynomials when suitable factorizations are available,	

and use the zeros to construct a rough graph of the function defined by the polynomial. (CCSS: A-APR.3) e. Use polynomial identities to solve problems. (CCSS: A-APR) i. Prove polynomial identities[18] and use them to describe numerical relationships. (CCSS: A-APR.4) f. Rewrite rational expressions. (CCSS: A-APR) g. Rewrite simple rational expressions in different forms.[19] (CCSS: A-APR.6)	

[13] For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P. (CCSS: A-SSE.1b)

[14] For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. (CCSS: A-SSE.2)

[15] For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. (CCSS: A-SSE.3c)

[16] For example, calculate mortgage payments. (CCSS: A-SSE.4)

[17] For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x). (CCSS: A-APR.2)

[18] For example, the polynomial identity $(x^2+y^2)^2=(x^2-y^2)^2+(2xy)^2$ can be used to generate Pythagorean triples. (CCSS: A-APR.4)

[19] write $a(x)b(x)$ in the form $q(x)+r(x)b(x)$, where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. (CCSS: A-APR.6)

Prepared Graduates: *(Click on a Prepared Graduate Competency to View Articulated Expectations)*

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Concepts and skills students master:

4. Solutions to equations, inequalities and systems of equations are found using a variety of tools

Evidence Outcomes	21st Century Skill and Readiness Competencies
Students Can: a. Create equations that describe numbers or relationships. (CCSS: A-CED) i. Create equations and inequalities[20] in one variable and use them to solve problems. (CCSS: A-CED.1) ii. Create equations in two or more variables to represent relationships between quantities and graph equations on coordinate axes with labels and scales. (CCSS: A-CED.2) iii. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.[21] (CCSS: A-CED.3)	**Inquiry Questions:** 1. What are some similarities in solving all types of equations? 2. Why do different types of equations require different types of solution processes? 3. Can computers solve algebraic problems that people cannot solve? Why? 4. How are order of operations and operational relationships important when solving multivariable equations? **Relevance & Application:**

iv. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.[22] (CCSS: A-CED.4)

b. Understand solving equations as a process of reasoning and explain the reasoning. (CCSS: A-REI)
   i. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. (CCSS: A-REI.1)
   ii. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. (CCSS: A-REI.2)

c. Solve equations and inequalities in one variable. (CCSS: A-REI)
   i. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. (CCSS: A-REI.3)
   ii. Solve quadratic equations in one variable. (CCSS: A-REI.4)
      1. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x-p)^2=q$ that has the same solutions. Derive the quadratic formula from this form. (CCSS: A-REI.4a)
      2. Solve quadratic equations[23] by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. (CCSS: A-REI.4b)
      3. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b. (CCSS: A-REI.4b)

d. Solve systems of equations. (CCSS: A-REI)
   i. Prove that, given a system of two equations in two

1. Linear programming allows representation of the constraints in a real-world situation identification of a feasible region and determination of the maximum or minimum value such as to optimize profit, or to minimize expense.
2. Effective use of graphing technology helps to find solutions to equations or systems of equations.

Nature Of:

1. Mathematics involves visualization.
2. Mathematicians use tools to create visual representations of problems and ideas that reveal relationships and meaning.
3. Mathematicians construct viable arguments and critique the reasoning of others. (MP)
4. Mathematicians use appropriate tools strategically. (MP)

variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. (CCSS: A-REI.5) ii. Solve systems of linear equations exactly and approximately,[24] focusing on pairs of linear equations in two variables. (CCSS: A-REI.6) iii. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.[25](CCSS: A-REI.7) e. Represent and solve equations and inequalities graphically. (CCSS: A-REI) i. Explain that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve.[26] (CCSS: A-REI.10) ii. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x);[27] find the solutions approximately.[28] * (CCSS: A-REI.11) iii. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. (CCSS: A-REI.12)	

[20] Include equations arising from linear and quadratic functions, and simple rational and exponential functions. (CCSS: A-CED.1)

[21] For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. (CCSS: A-CED.3)

[22] For example, rearrange Ohm's law V = IR to highlight resistance R. (CCSS: A-CED.4)

[23] e.g., for $x^2 = 49$. (CCSS: A-REI.4b)

[24] e.g., with graphs. (CCSS: A-REI.6)

[25] For example, find the points of intersection between the line y = –3x and the circle $x^2 + y^2 = 3$. (CCSS: A-REI.7)

[26] which could be a line. (CCSS: A-REI.10)

[27] Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. (CCSS: A-REI.11)

[28] e.g., using technology to graph the functions, make tables of values, or find successive approximations. (CCSS: A-REI.11)

Content Area: Mathematics

Grade Level Expectations: High School

Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates: *(Click on a Prepared Graduate Competency to View Articulated Expectations)* • Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data
Concepts and skills students master: 1. Visual displays and summary statistics condense the information in data sets into usable knowledge

Evidence Outcomes	21st Century Skill and Readiness Competencies

Students Can:
 	
a. Summarize, represent, and interpret data on a single count or measurement variable. (CCSS: S-ID)
  i. Represent data with plots on the real number line (dot plots, histograms, and box plots). (CCSS: S-ID.1)
  ii. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. (CCSS: S-ID.2)
  iii. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). (CCSS: S-ID.3)
  iv. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages and identify data sets for which such a procedure is not appropriate. (CCSS: S-ID.4)
  v. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. (CCSS: S-ID.4)
b. Summarize, represent, and interpret data on two categorical and quantitative variables. (CCSS: S-ID)
  i. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data[1]
	Inquiry Questions:
	
	1. What makes data meaningful or actionable?
	2. Why should attention be paid to an unexpected outcome?
	3. How can summary statistics or data displays be accurate but misleading?
	Relevance & Application:
	
	1. Facility with data organization, summary, and display allows the sharing of data efficiently and collaboratively to answer important questions such as is the climate changing, how do people think about ballot initiatives in the next election, or is there a connection between cancers in a community?
	Nature Of:
	
	1. Mathematicians create visual and numerical representations of data to reveal relationships and meaning hidden in the raw data.
	2. Mathematicians reason abstractly and quantitatively. (MP)
	3. Mathematicians model with mathematics. (MP)
	4. Mathematicians use appropriate tools strategically. (MP)

(including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. (CCSS: S-ID.5) ii. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. (CCSS: S-ID.6) 1. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. (CCSS: S-ID.6a) 2. Informally assess the fit of a function by plotting and analyzing residuals. (CCSS: S-ID.6b) 3. Fit a linear function for a scatter plot that suggests a linear association. (CCSS: S-ID.6c) c. Interpret linear models. (CCSS: S-ID) i. Interpret the slope[2] and the intercept[3] of a linear model in the context of the data. (CCSS: S-ID.7) ii. Using technology, compute and interpret the correlation coefficient of a linear fit. (CCSS: S-ID.8) iii. Distinguish between correlation and causation. (CCSS: S-ID.9)	

[1] including joint, marginal, and conditional relative frequencies.

[2] rate of change. (CCSS: S-ID.7)

[3] constant term. (CCSS: S-ID.7)

Prepared Graduates: *(Click on a Prepared Graduate Competency to View Articulated Expectations)* ● Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking
Concepts and skills students master: 2. Statistical methods take variability into account supporting informed decisions making through quantitative studies designed to answer specific questions

Evidence Outcomes	**21st Century Skill and Readiness Competencies**
Students Can: a. Understand and evaluate random processes underlying statistical experiments. (CCSS: S-IC) i. Describe statistics as a process for making inferences about population parameters based on a random sample from that population. (CCSS: S-IC.1) ii. Decide if a specified model is consistent with results from a given data-generating process.[4] (CCSS: S-IC.2) b. Make inferences and justify conclusions from sample surveys, experiments, and observational	**Inquiry Questions:** 1. How can the results of a statistical investigation be used to support an argument? 2. What happens to sample-to-sample variability when you increase the sample size? 3. When should sampling be used? When is sampling better than using a census? 4. Can the practical significance of a given study matter more than statistical significance? Why is it important to know the difference? 5. Why is the margin of error in a study important? 6. How is it known that the results of a study are not simply due

studies. (CCSS: S-IC) i. Identify the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. (CCSS: S-IC.3) ii. Use data from a sample survey to estimate a population mean or proportion. (CCSS: S-IC.4) iii. Develop a margin of error through the use of simulation models for random sampling. (CCSS: S-IC.4) iv. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. (CCSS: S-IC.5) v. Define and explain the meaning of significance, both statistical (using p-values) and practical (using effect size). vi. Evaluate reports based on data. (CCSS: S-IC.6)	to chance? **Relevance & Application:** 1. Inference and prediction skills enable informed decision-making based on data such as whether to stop using a product based on safety concerns, or whether a political poll is pointing to a trend. **Nature Of:** 1. Mathematics involves making conjectures, gathering data, recording results, and making multiple tests. 2. Mathematicians are skeptical of apparent trends. They use their understanding of randomness to distinguish meaningful trends from random occurrences. 3. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 4. Mathematicians model with mathematics. (MP) 5. Mathematicians attend to precision. (MP)

[4] e.g., using simulation. (CCSS: S-IC.2)

For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model? (CCSS: S-IC.2)

Prepared Graduates: *(Click on a Prepared Graduate Competency to View Articulated Expectations)*

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts

Concepts and skills students master: 3. Probability models outcomes for situations in which there is inherent randomness	
Evidence Outcomes	**21st Century Skill and Readiness Competencies**
Students Can: a. Understand independence and conditional probability and use them to interpret data. (CCSS: S-CP) i. Describe events as subsets of a sample space[5] using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events.[6] (CCSS: S-CP.1) ii. Explain that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. (CCSS: S-CP.2) iii. Using the conditional probability of A given B as P(A and B)/P(B), interpret the independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. (CCSS: S-CP.3) iv. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional	**Inquiry Questions:** 1. Can probability be used to model all types of uncertain situations? For example, can the probability that the 50th president of the United States will be female be determined? 2. How and why are simulations used to determine probability when the theoretical probability is unknown? 3. How does probability relate to obtaining insurance? (PFL) **Relevance & Application:** 1. Comprehension of probability allows informed decision-making, such as whether the cost of insurance is less than the expected cost of illness, when the deductible on car insurance is optimal, whether gambling pays in the long run, or whether an extended warranty justifies the cost. (PFL) 2. Probability is used in a wide variety of disciplines including physics, biology, engineering, finance, and law. For example, employment discrimination cases often present probability calculations to support a claim.

probabilities.[7] (CCSS: S-CP.4) v. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.[8] (CCSS: S-CP.5) b. Use the rules of probability to compute probabilities of compound events in a uniform probability model. (CCSS: S-CP) i. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. (CCSS: S-CP.6) ii. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model. (CCSS: S-CP.7) c. Analyze the cost of insurance as a method to offset the risk of a situation. (PFL)	**Nature Of:** 1. Some work in mathematics is much like a game. Mathematicians choose an interesting set of rules and then play according to those rules to see what can happen. 2. Mathematicians explore randomness and chance through probability. 3. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 4. Mathematicians model with mathematics. (MP)

[5] the set of outcomes. (CCSS: S-CP.1)

[6] "or," "and," "not". (CCSS: S-CP.1)

[7] For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. (CCSS: S-CP.4)

[8] For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. (CCSS: S-CP.5)

Content Area: Mathematics

Grade Level Expectations: High School

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates: *(Click on a Prepared Graduate Competency to View Articulated Expectations)*

- Apply transformation to numbers, shapes, functional representations, and data

Concepts and skills students master:

1. Objects in the plane can be transformed, and those transformations can be described and analyzed mathematically

Evidence Outcomes	21st Century Skill and Readiness Competencies

Students Can:
 	
a. Experiment with transformations in the plane. (CCSS: G-CO)
   i. State precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. (CCSS: G-CO.1)
   ii. Represent transformations in the plane using[1] appropriate tools. (CCSS: G-CO.2)
   iii. Describe transformations as functions that take points in the plane as inputs and give other points as outputs. (CCSS: G-CO.2)
   iv. Compare transformations that preserve distance and angle to those that do not.[2](CCSS: G-CO.2)
   v. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry
	Inquiry Questions:
	
	1. What happens to the coordinates of the vertices of shapes when different transformations are applied in the plane?
	2. How would the idea of congruency be used outside of mathematics?
	3. What does it mean for two things to be the same? Are there different degrees of "sameness?"
	4. What makes a good definition of a shape?
	Relevance & Application:
	
	1. Comprehension of transformations aids with innovation and creation in the areas of computer graphics and animation.

it onto itself. (CCSS: G-CO.3) vi. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. (CCSS: G-CO.4) vii. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using appropriate tools.[3](CCSS: G-CO.5) viii. Specify a sequence of transformations that will carry a given figure onto another. (CCSS: G-CO.5) b. Understand congruence in terms of rigid motions. (CCSS: G-CO) i. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure. (CCSS: G-CO.6) ii. Given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. (CCSS: G-CO.6) iii. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. (CCSS: G-CO.7) iv. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. (CCSS: G-CO.8) c. Prove geometric theorems. (CCSS: G-CO) i. Prove theorems about lines and angles.[4](CCSS: G-CO.9) ii. Prove theorems about triangles.[5] (CCSS: G-CO.10) iii. Prove theorems about parallelograms.[6](CCSS: G-CO.11) d. Make geometric constructions. (CCSS: G-CO) i. Make formal geometric constructions[7] with a variety of tools and methods.[8] (CCSS: G-CO.12) ii. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. (CCSS: G-CO.13)	**Nature Of:** 1. Geometry involves the investigation of invariants. Geometers examine how some things stay the same while other parts change to analyze situations and solve problems. 2. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 3. Mathematicians attend to precision. (MP) 4. Mathematicians look for and make use of structure. (MP)

Prepared Graduates: *(Click on a Prepared Graduate Competency to View Articulated Expectations)*

- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Concepts and skills students master:

2. Concepts of similarity are foundational to geometry and its applications

Evidence Outcomes	21st Century Skill and Readiness Competencies

Students Can:
 	
a. Understand similarity in terms of similarity transformations. (CCSS: G-SRT)
  i. Verify experimentally the properties of dilations given by a center and a scale factor. (CCSS: G-SRT.1)
    1. Show that a dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. (CCSS: G-SRT.1a)
    2. Show that the dilation of a line segment is longer or shorter in the ratio given by the scale factor. (CCSS: G-SRT.1b)
  ii. Given two figures, use the definition of
	Inquiry Questions:
	
	1. How can you determine the measure of something that you cannot measure physically?
	2. How is a corner square made?
	3. How are mathematical triangles different from triangles in the physical world? How are they the same?
	4. Do perfect circles naturally occur in the physical world?
	
	Relevance & Application:
	
	1. Analyzing geometric models helps one understand complex physical systems. For example, modeling Earth as a sphere allows us to calculate measures such as diameter, circumference, and surface area. We can also model the solar system, galaxies, molecules, atoms, and subatomic particles.
	
	Nature Of:
	
	1. Geometry involves the generalization of ideas. Geometers seek to

similarity in terms of similarity transformations to decide if they are similar. (CCSS: G-SRT.2) iii. Explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. (CCSS: G-SRT.2) iv. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. (CCSS: G-SRT.3) b. Prove theorems involving similarity. (CCSS: G-SRT) i. Prove theorems about triangles.[9] (CCSS: G-SRT.4) ii. Prove that all circles are similar. (CCSS: G-C.1) iii. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. (CCSS: G-SRT.5) c. Define trigonometric ratios and solve problems involving right triangles. (CCSS: G-SRT) i. Explain that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. (CCSS: G-SRT.6) ii. Explain and use the relationship	understand and describe what is true about all cases related to geometric phenomena. 2. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 3. Mathematicians attend to precision. (MP)

between the sine and cosine of complementary angles. (CCSS: G-SRT.7)

iii. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.* (CCSS: G-SRT.8)

d. Prove and apply trigonometric identities. (CCSS: F-TF)

i. Prove the Pythagorean identity $sin^2(\theta)+cos^2(\theta)=1$. (CCSS: F-TF.8)

ii. Use the Pythagorean identity to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle. (CCSS: F-TF.8)

e. Understand and apply theorems about circles. (CCSS: G-C)

i. Identify and describe relationships among inscribed angles, radii, and chords.[10](CCSS: G-C.2)

ii. Construct the inscribed and circumscribed circles of a triangle. (CCSS: G-C.3)

iii. Prove properties of angles for a quadrilateral inscribed in a circle. (CCSS: G-C.3)

f. Find arc lengths and areas of sectors of circles. (CCSS: G-C)

i. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle

as the constant of proportionality. (CCSS: G-C.5) ii. Derive the formula for the area of a sector. (CCSS: G-C.5)	

[9] Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. (CCSS: G-SRT.4)

[10] Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. (CCSS: G-C.2)

Prepared Graduates: *(Click on a Prepared Graduate Competency to View Articulated Expectations)*

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Concepts and skills students master:

3. Objects in the plane can be described and analyzed algebraically

Evidence Outcomes	21st Century Skill and Readiness Competencies
Students Can: a. Express Geometric Properties with Equations. (CCSS: G-GPE) i. Translate between the geometric description and the equation for a	**Inquiry Questions:** 1. What does it mean for two lines to be parallel? 2. What happens to the coordinates of the vertices of shapes when different transformations are applied in the plane?

conic section. (CCSS: G-GPE) 1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem. (CCSS: G-GPE.1) 2. Complete the square to find the center and radius of a circle given by an equation. (CCSS: G-GPE.1) 3. Derive the equation of a parabola given a focus and directrix. (CCSS: G-GPE.2) ii. Use coordinates to prove simple geometric theorems algebraically. (CCSS: G-GPE) 1. Use coordinates to prove simple geometric theorems[11] algebraically. (CCSS: G-GPE.4) 2. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems.[12] (CCSS: G-GPE.5) 3. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. (CCSS: G-GPE.6) 4. Use coordinates and the distance formula to compute perimeters of polygons and areas of triangles and	**Relevance & Application:** 1. Knowledge of right triangle trigonometry allows modeling and application of angle and distance relationships such as surveying land boundaries, shadow problems, angles in a truss, and the design of structures. **Nature Of:** 1. Geometry involves the investigation of invariants. Geometers examine how some things stay the same while other parts change to analyze situations and solve problems. 2. Mathematicians make sense of problems and persevere in solving them. (MP) 3. Mathematicians construct viable arguments and critique the reasoning of others. (MP)

rectangles.* (CCSS: G-GPE.7)	

[11] For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, 3√) lies on the circle centered at the origin and containing the point (0, 2). (CCSS: G-GPE.4)

[12] e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point. (CCSS: G-GPE.5)

Prepared Graduates: *(Click on a Prepared Graduate Competency to View Articulated Expectations)* • Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics	
Concepts and skills students master: 4. Attributes of two- and three-dimensional objects are measurable and can be quantified	
Evidence Outcomes	**21st Century Skill and Readiness Competencies**
Students Can: a. Explain volume formulas and use them to solve problems. (CCSS: G-GMD) i. Give an informal argument[13] for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. (CCSS: G-GMD.1) ii. Use volume formulas for cylinders,	**Inquiry Questions:** 1. How might surface area and volume be used to explain biological differences in animals? 2. How is the area of an irregular shape measured? 3. How can surface area be minimized while maximizing volume? **Relevance & Application:** 1. Understanding areas and volume enables design and building. For example, a container that maximizes volume and minimizes

pyramids, cones, and spheres to solve problems.* (CCSS: G-GMD.3) b. Visualize relationships between two-dimensional and three-dimensional objects. (CCSS: G-GMD) i. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. (CCSS: G-GMD.4)	surface area will reduce costs and increase efficiency. Understanding area helps to decorate a room, or create a blueprint for a new building. **Nature Of:** 1. Mathematicians use geometry to model the physical world. Studying properties and relationships of geometric objects provides insights in to the physical world that would otherwise be hidden. 2. Mathematicians make sense of problems and persevere in solving them. (MP) 3. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 4. Mathematicians model with mathematics. (MP)

[13] Use dissection arguments, Cavalieri's principle, and informal limit arguments. (CCSS: G-GMD.1)

Prepared Graduates: *(Click on a Prepared Graduate Competency to View Articulated Expectations)*

- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Concepts and skills students master:

5. Objects in the real world can be modeled using geometric concepts

Evidence Outcomes	21st Century Skill and Readiness Competencies

Students Can:	Inquiry Questions:
a. Apply geometric concepts in modeling situations. (CCSS: G-MG) i. Use geometric shapes, their measures, and their properties to describe objects.[14] * (CCSS: G-MG.1) ii. Apply concepts of density based on area and volume in modeling situations.[15] * (CCSS: G-MG.2) iii. Apply geometric methods to solve design problems.[16] * (CCSS: G-MG.3)	1. How are mathematical objects different from the physical objects they model? 2. What makes a good geometric model of a physical object or situation? 3. How are mathematical triangles different from built triangles in the physical world? How are they the same? **Relevance & Application:** 1. Geometry is used to create simplified models of complex physical systems. Analyzing the model helps to understand the system and is used for such applications as creating a floor plan for a house, or creating a schematic diagram for an electrical system. **Nature Of:** 1. Mathematicians use geometry to model the physical world. Studying properties and relationships of geometric objects provides insights in to the physical world that would otherwise be hidden. 2. Mathematicians make sense of problems and persevere in solving them. (MP) 3. Mathematicians reason abstractly and quantitatively. (MP) 4. Mathematicians look for and make use of structure. (MP)

[14] e.g., modeling a tree trunk or a human torso as a cylinder. (CCSS: G-MG.1)

[15] e.g., persons per square mile, BTUs per cubic foot. (CCSS: G-MG.2)

[16] e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios. (CCSS: G-MG.3)